International Journal of Engineering, Science and Mathematics
Vol. 7Issue 1, January2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

DIFFUSION APPROXIMATION TO MULTI SERVER QUEUE WITH DISCOURAGEMENT AND BULK ARRIVAL

Dr. Sadhna Golash[1], Dr. G.C. Sharma[2]
[1-2]Department of Mathematics, Institute of Basic Science Khandari, Agra

Abstract

This paper is devoted to present the simple approximate results for the $M^{\alpha}/M/C$ queuing system, applying the method of diffusion approximation for the transient behavior of probability density for the system state $P(X,\ t/X_0 = X_0)$, using the method of diffusion approximation. We provide approximate modified formulae for the distributions of the mean number of customers and waiting time.

Keywords:
Diffusion approximation, transient behavior, Discouragement, Steady state.

Author correspondence:
Dr. Sadhna Golash
Institute of Basic Science, Khandari, Agra (India)

1. **Inroduction:** In real life these are many queueing situations in which long queue may tend to allow the arrival stream by discouraging potential customers from entering. Several researchers have proposed single server queueing models in which the arrival rate is decreasing function of the queue length. Anker and Gafarian (1963 a) find a queueing situation in which the arrival rate falls linearly to zero when the queue reaches to a certain size. Anker and Gafarian (1963 b) and Cox and Smith (1961) studied the single channel model in which he arrival rate is inversely proportional to the number of customers in the system.

Reynods (1968) and Cox and Smith (1968) have proposed approximate formule for the multi-server queueing system. They study M/M/C model with discouragement. Also they modified he exact result for M/M/C systems in terms of each first two moments of service time distribution. Some authors have also suggested a mean waiting time formula for the M/G/M system by combining the exact results of the M/M/m and M/D/m systems.

In recent last Baburaj and Manoharan (2004), studied the M/G/I queueing system. Alfa (2003) analyses $M^{KA}/G/I$ System in discrete time. Antunes at el (2006) did. Perturbation analysis of a variable M/M/I queue. We consider the model $M^{\infty}/M/C$ in which the arrival rate is a decreasing function of the queue length. For the steady state, by applying the method of diffusion approximation, we provide approximate formula for the mean number of customers in the system and the mean waiting time for the customers.

Amari, Kihl and Robertsson (2011) said that multi-step ahead response time prediction for single server queueing systems. Amiri, Pedarsari, Skaboudoxis and Varaiya (2016) studied that queue-length estimation using real-time traffic data.

We consider the model $M^{\alpha}/M/C$ in which the arrival rate is a decreasing function of the queue length. For the steady state, by applying the method of diffusion approximation, we provide approximate formula for the mean number of customers in the system and the mean waiting time for the customers.

On the basis of the heavy traffic limit theorems, some authors approximated he discrete valued process of the number of customers by a diffusion process with negative drift and gave an approximate distribution of the number of customers in the system. This method is called a diffusion approximation.

1. Model and its solution by diffusion approximation method:

We consider an $M^{\alpha}/M/C$ queue with state dependent parameter $\lambda\alpha\eta$ and μ_{η} given by

$$\lambda\alpha\eta = \begin{cases} \lambda\alpha & (n < c) \\ \lambda\alpha/(n-c+2) & (n \geq c) \end{cases} \quad (1)$$

$$\mu_{\eta} = \begin{cases} \eta\mu & (n \leq c) \\ c\mu & (n \geq c) \end{cases} \quad (2)$$

where n is the number in the system and α is the number of arrivals in batches.

This multi-server queueing model has arrival rate $\lambda\alpha$ as long as there is at least one free server, but as soon as all servers are busy, the arrival rate is inversely proportional to $(n - c + 2)$. Then the balking probability in $M^{\alpha}/M/C$ is $1 - 1/(n - c + 2)$ for $n \geq c$ and zero for $n < c$. the service times in this model is exponentially distributed with mean service rate is μ.

Let

Y (t) – Number of customers in the system at time t.

Pn(t) – Probability that n customers in the system at time t.

In the method of diffusion approximation, we replace the discrete variable Y(t) by a continuous variable X(t). We consider that X(t) has a density function (P(X, t) defined as

P(X, t) dx = Prob. { x ≤ x ≤ x + dx }

We consider the steady state solution, Let

Pn = $\lim_{t-->\infty} P_n\ (t)$ and

P(x) = $\lim_{t-->\infty} P(x,t)$

The P(X) satisfies the forward Kolmogorov equation (Cox and Miller 1965)

$$\frac{\partial P}{\partial t} = -\frac{\partial[a(x)P\ (x)]}{\partial x} + \frac{1}{2}\partial^2[b(x)P\ (x)]/\partial x^2 \qquad (3)$$

Where, a (x) is the infinitesimal mean of the conditional expectation of the increment of X(t) and it's also knows as the drift coefficient, defined as

$$a(x) = \lim_{t-->0} E\{X(t+t) - X\ (t)|\ \ X(t) - x\}/t \qquad (4)$$

And b(x) is the infinitesimal mean of the variances of the increment of X(t) and it's called the diffusion coefficient, defined as

$$b(x) = \lim_{t-->0} E\{[X(t+t) - X\ (t)]^2|\ \ X(t) - x\}/t \qquad (5)$$

For our queueing system, we propose (Halachami and Franta, 1978)

$$a(x) = \begin{cases} \lambda\alpha - x\mu & (x < c) \\ \lambda\alpha/[(x-c+2) - c\ \mu] & (x \geq c) \end{cases} \qquad (6)$$

$$b(x) = \begin{cases} \lambda\alpha - x\mu & (x < c) \\ \lambda\alpha/[(x-c+2) + c\ \mu] & (x \geq c) \end{cases} \qquad (7)$$

By integrating the equation (3), we get

$$\frac{1}{2}\frac{\partial[b(x)P\ (x)]}{\partial x} - a(x)\ P\ (x) = 0 \qquad (8)$$

Then

$$P(x) = \frac{c}{b(x)}\ \ e \times p\{2\ \int_0^x \{a(x)|b(x)\}\ \ dx] \qquad (9)$$

Where, C is the constant of integration.

This equation has two solution $P_1(x)$ and $P_2(x)$ defined as follows :

For x < c

$$P_1(x) = c(\lambda\alpha + x\mu)^{\left[\frac{4\lambda\alpha}{\mu}-1\right]}e^{-2x} = cq_1(x) \qquad (10)$$

For x ≥ c, from the continuity of P(x) at x = c

$$P_2(x) = c\{\frac{q_1(c)}{q_2(c)}\}\quad q_2(x) \qquad (11)$$

Where $q_2(x) = \{\frac{\lambda\alpha}{x-c+2} - c\mu)^{\left[\frac{4\lambda\alpha}{c\mu}-1\right]}(x-c+2)^{\frac{4\lambda\alpha}{c\mu}}e^{-2x}$

The integrating constant c can be determined by using normalizing condition that the integrated value of P(x) over the region (0, ∞) is unity, i.e.

$\int_0^c P_1(x)\,dx + \int_c^\infty P_{2(x)\,dx} = 1$ (12)

The probability that the customers have to wait is given by

$\pi = \int_{c-1}^\infty P\,(x)dx$ (13)

3. Queue Length

The mean number of customers in the queue is obtained by using the formula.

^

sq = $\int_c^\infty (x-c)$ $P_2(x)\,dx$ (14)

The approximate formula for the mean number of customers in the system by using continuous probability density function (P (x) as

^

Ec (Q) = $\int_0^\infty xP(x)dx)$ (15)

whereEc(q) is the mean and variance.

We formulate a modified formula for the mean number of customers. Define it

(Q) = $\int_0^\infty x^2 \quad P(x)\,dx$

We know the usual result for M^α/ M/1 system, i.e.

P (x) = $\mu_j\lambda\alpha/2\,(\mu-\lambda\alpha)\exp\{-[2\,(\mu-\lambda\alpha)/\mu+\lambda\alpha]\}\,x$

Put his value in equation (15), we get

Ec (Q) = $(c\mu+\lambda\alpha)^3/8\,(c\mu-\lambda\alpha)^3$

And Var (Q) = $(c\mu+\lambda\alpha)^4/8\,(c\mu-\lambda\alpha)^4$

We formulate a modified formula for the mean number of customers. Define it

$$E(Q) = (1 - C_{W^2})\ E(Q) + C_{W^2} E_W(Q)$$

Where, C_W is the coefficient of variation of service times, E(w) and E(q) is the mean number of customers in steady state and given by

$$E(Q) = P\{(1 + P(1 + C_{W^2})\,/2\ \ (1 - P)\}$$

$$E(w) = (1/\lambda\alpha)\,[E(Q) - Cp]$$

4. Numerical Example

The accuracy of the fluid approximation for the queue length process were considered in [Mandelbaum et al,5]. This example had constant arrival rate, exhibited the approach to equilibrium, and the fluid approximation was excellent.

Here we examine the performance of the fluid and diffusion approximation for both queue length and virtual waiting time. We merely point out that we use 5000 independent application in our experiment. By contrast, all the fluid and diffusion approximation used here from numerically integrating 7 ordinary differential equations.

Here we cover the case of time varying behaviour only for the external arrival rate λ_t. The type of time varying behavior used is that of a periodic square wave, oscillating between two values (starting with the smaller value) and the duration of each value is 2 time units for a total time interval of 20 time units.

The graphs are ordered by pairing the 20/100 case first (the top graph) followed by the 40/80 case (the bottom graph) for the numerical plot :

Empirical averages of $Q_1(t)$ and $Q_2(t)$ versus their fluid approximations.

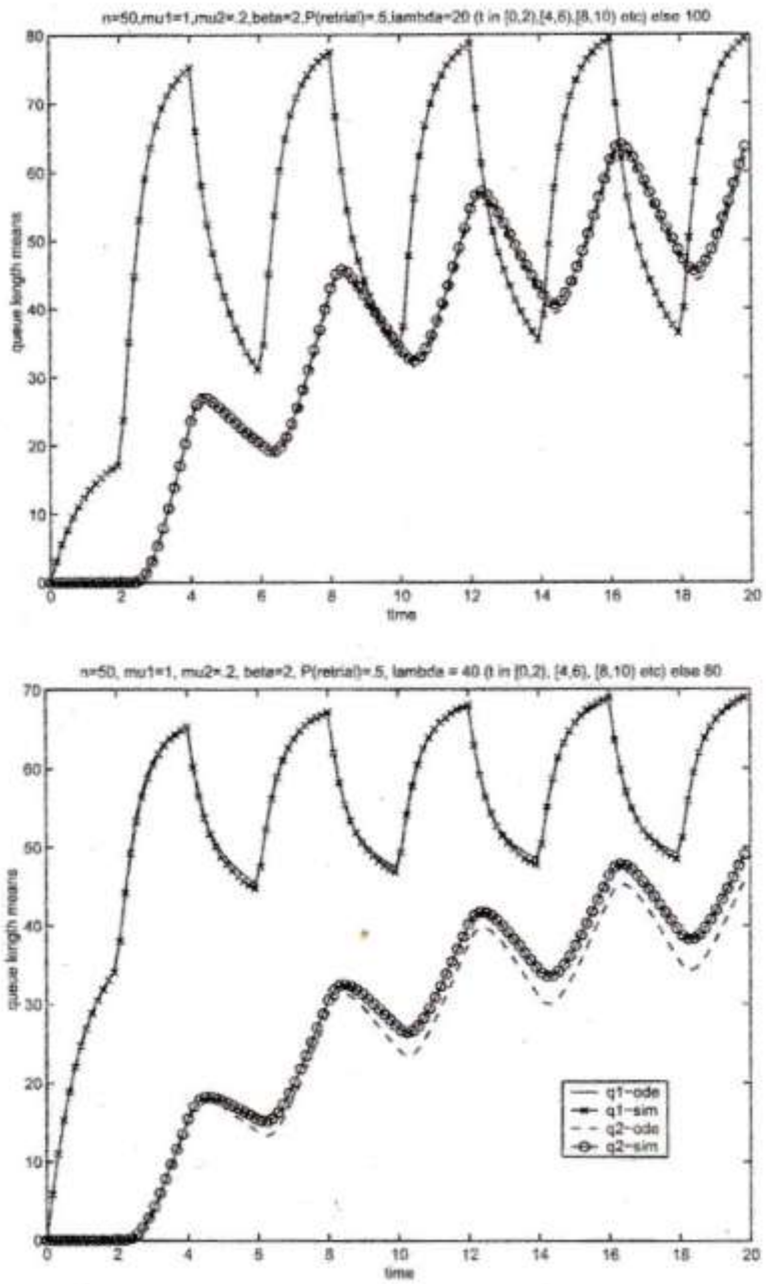

Figure 2. Numerical example: Empirical averages of $Q_1(t)$ and $Q_2(t)$ versus their fluid approximations for the 20/100 and 40/80 square wave cases.

Conclusions:

We have derived an approximate modified formula for the distribution of the mean number of customers and waiting time applying diffusion approximation method.

References:

[1] AldousjD, Miyazawa, M, and Rdski, T (2001) on the stability of a batch clearing system with poisson arrivals and subadditiveserive times, J. Appl, Probab. 38, no. 3, 621-634.

[2] Al-Jararha, J. and Madan, K.C.,)2002) Steady State analysis of aji M/D/I queue with coxian – 2 server vacation and a single vacation policy. International Journal of Information and Management Sciences, Vol. 13, No 4, PP 69 – 81.

[3] Alfa, A.S. (2003), M^A/G/I queue – setup time vacation period – busy period and queue size vacation models in discrete time, queueing systems, volume 44, Number 1, 5 – 30.

[4] Baburaj, C. and Manoharan, M. (2004), on the optimal control limit of a single and batch service M/G/I queue, JISA, Vol. 42, PP 49-61.

[5] Alfa, A.S. (2005), Analysis of a GI/G^A/I system in discrete time, Stochastic Models, 21 (1) : 185 – 213.

[6] Antunes, Nelson, Pricker, Chirstine, Guillemin, Fabrice, and Robert, Phiuppe (2006) Perturbation analysis of a variable M/M/I queue a probabilistic approach. Adv. April, Probab, 38, no1, 263 – 283.

[7] P. Amari, M. Kihl, and A. Robertsson, Multi-Step ahead response time prediction for single server queueing systems in computers and communications (ISCC), 2011 IEEE symposium on : 850 – 955, IEEE 2011.

[8] Z. Amiri, R. Pedarsari, A. Skabardaris, and P. Varaiya, Queue-length estimation using real time traffic data. In intelligent transportion systems (ITSC), 2016 IEEE 19th International conference on IEEE : 1476 – 1481, 2016.